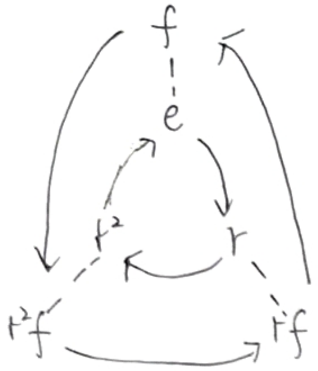
D_3群 凯莱图

r: 120°旋转

r^2: 240°旋转

f: 水平翻转

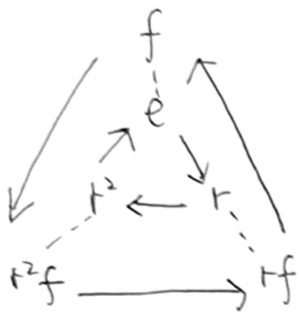


乘法表.

$D_3 = \langle r, f \mid r^3 = e, f^2 = e, rf = fr^2 \rangle$

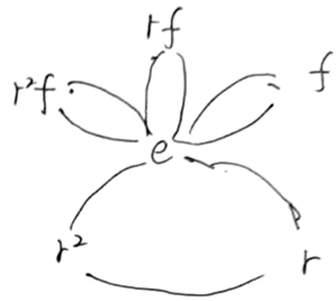
$= \langle r, f \mid r^3 = e, f^2 = e, rfr = f \rangle$

从右往左作用: $r^2 f \xrightarrow{r} r^2 f r = rf \xrightarrow{r} rfr = f$



群元	轨道
e	$\{e\}$
r	$\{e, r, r^2\}$
r^2	$\{e, r^2, r\}$
f	$\{e, f\}$
rf	$\{e, rf\}$
r^2f	$\{e, r^2f\}$

轨道图 →



g的轨道：作用g与g^{-1}任意多次得到的集合. $\langle g \rangle = \{g^k : k \in \mathbb{Z}\}$

C_n: 循环群，描述2维多面体的旋转对称性.

D_n: 二面体群. 描述2维多面体的转动对称性 + 反射对称性.

D_n生成元: r (顺时针转 $\frac{2\pi}{n}$) + f (绕某个轴转)

用相邻替换写出S_3的所有元素

e: 1, 2, 3

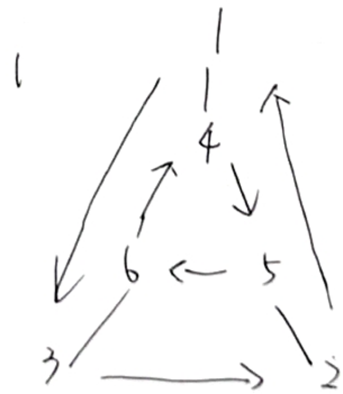
f: 1, 3, 2 $\quad (2,3)$

r: 2, 3, 1 $\quad (1,2)(2,3)$

r^2: 3, 1, 2 $\quad (1,2)(2,3)(1,2)(2,3)$

rf: 2, 1, 3 $\quad (1,2)$

r^2f: 3, 2, 1 $\quad (1,3) = (1,2)(2,3)(1,2)$

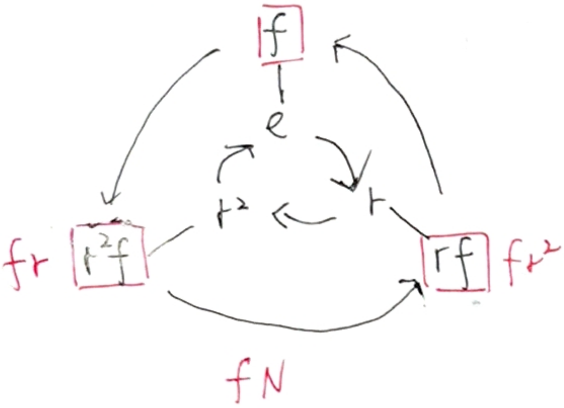


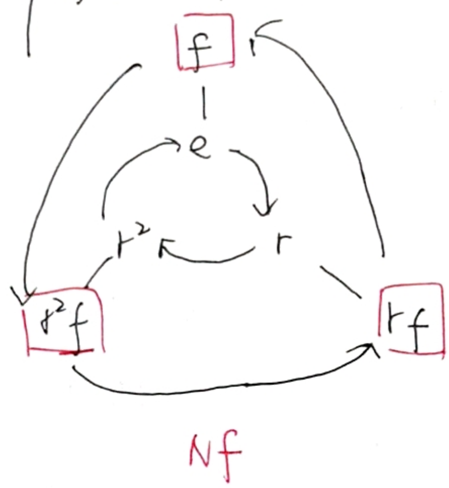
$D_3 \simeq \langle (132)(456), (14)(25)(36) \rangle < S_6$

正规子群:

D_3的子群 $N=\{e,r,r^2\}\subset D_3$ 是否为正规子群?

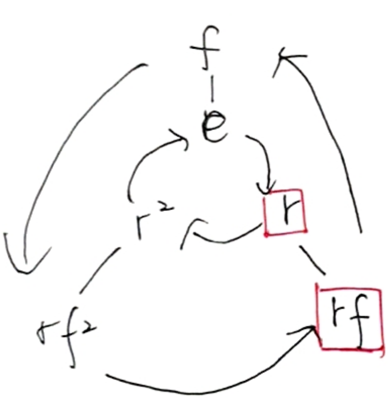
- N的指数 $[G:H]=2 \Rightarrow$ N为正规子群.
- $\left.\begin{aligned} fN &= f\{e,r,r^2\} = \{f,fr,fr^2\} \\ Nf &= \{e,r,r^2\}f = \{f,rf,r^2f\} \end{aligned}\right\} \Rightarrow fN=Nf$ 即 N为正规子群.

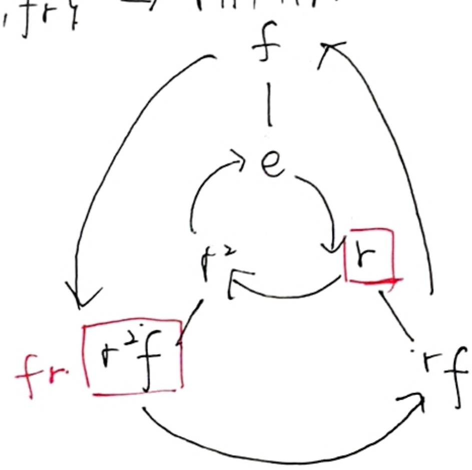




D_3的子群 $H=\langle f\rangle=\{e,f\}$

- $rH=\{r,rf\}$. $Hr=\{r,fr\}$ $\Rightarrow rH\neq Hr$.

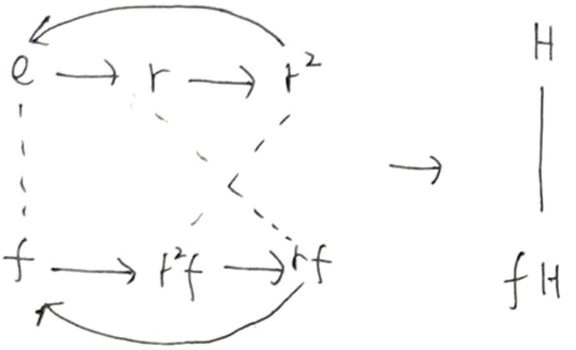




- $[G:H]=\dfrac{G}{H}=\dfrac{6}{2}=3\neq 2$.

$C_3 < D_3$

取 $G=D_3$, $H=\langle r\rangle \simeq C_3$. 有两个左陪集 $H=\{e,r,r^2\}$ 与 $fH=\{f,rf,r^2f\}$.



子群H在G中的正规化子:

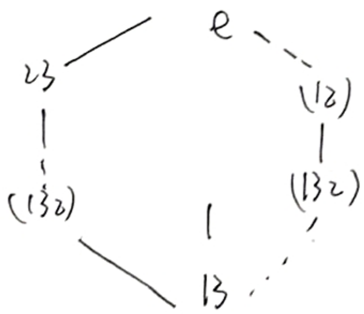
$N_G(H)=\{g\in G: gH=Hg\}=\{g\in G: gHg^{-1}=H\}$

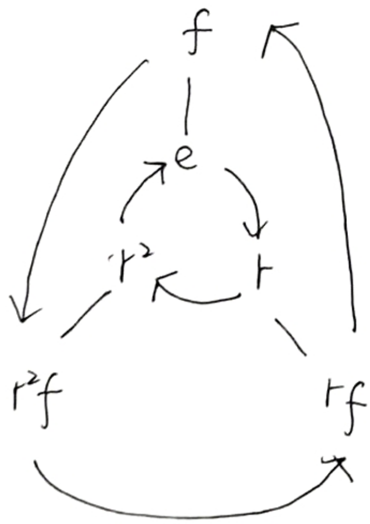
同构例子: $S_3\simeq D_3$.

$\phi: S_3 \to D_3$

$(12) \mapsto r^2f$

$(23) \mapsto f$





D_3 的自同构群 (332)

$D_3=\langle r,f\rangle$.

$$\begin{cases}\phi(e)=e\\ \phi(r)=\{r,r^2\}\\ \phi(f)=\{f,rf,r^2\}\end{cases}$$

$(\alpha): D_3\to D_3$ 固定 r $\begin{cases}\alpha(r)=r\\ \alpha(f)=rf\end{cases}$

$(\beta): D_3\to D_3$ 固定 f: $\begin{cases}\beta(r)=r^2\\ \beta(f)=f\end{cases}$

$\mathrm{Aut}(D_3)\simeq D_3$

李